

TLCA List of Open Problems

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Problem # 1 [SOLVED]

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Date: Known since 1958!

Statement. Is there a direct proof of the confluence of $\beta\eta$ -strong reduction?

Problem Origin. First posed by Haskell Curry and Roger Hindley.

The $\beta\eta$ -strong reduction is the combinatory analogue of $\beta\eta$ -reduction in λ -calculus. It is confluent. Its only known confluence-proof is very easy, [Curry and Feys, 1958, § 6F, p. 221 Theorem 3], but it depends on the having already proved the confluence of $\lambda\beta\eta$ -reduction. Thus the theory of combinators is not self-contained at present. Is there a confluence proof independent of λ -calculus?

Warning: Like a bog, the theory of strong reduction is messier than it looks at first; see [Curry and Feys, 1958, § 6F], [Curry et al., 1972, § 11E], [Hindley and Seldin, 1986, Ch. 9], and sources cited therein.

Solution: Two independent confluence proofs have been proposed in June 2008: one by René David [David, 2009], the other by Pierluigi Minari [Minari, 2009].

References

- [Curry and Feys, 1958] Curry, H. B. and Feys, R. (1958). *Combinatory Logic, Volume I*. North-Holland, Amsterdam. (3rd edition 1974).
- [Curry et al., 1972] Curry, H. B., Hindley, J. R., and Seldin, J. P. (1972). *Combinatory Logic, Volume II*. North-Holland, Amsterdam.
- [David, 2009] David, R. (2009). A direct proof of the confluence of combinatory strong reduction. *Theoretical Computer Science*, 410(42):4204–4215.
- [Hindley and Seldin, 1986] Hindley, J. R. and Seldin, J. P. (1986). *Introduction to Combinators and λ -calculus*. Cambridge University Press.
- [Minari, 2009] Minari, P. (2009). A solution to Curry and Hindley’s problem on combinatory strong reduction. *Archive for Mathematical Logic*, 43(2):159–184.