

TLCA List of Open Problems

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Problem # 11

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Statement. Which equations on intersection types preserve normalization?

Problem Origin. Included in the list of open problems from the “Gentzen” meeting in 1993. Author unknown.

Let E be a finite set of equations between intersection types, and let $=_E$ be the congruence generated by E . Extend intersection type assignment by the rule:

$$\frac{\Gamma \vdash M : \tau}{\Gamma \vdash M : \sigma} (\tau =_E \sigma)$$

This generalizes the notion of a recursive type introduced via “type constraints” in the style of [Mendler, 1991].

Under what condition such a type assignment system has the (strong) normalization property? As an example consider $E = \{a_1 = (a_2 \rightarrow a_1) \wedge (a_1 \rightarrow a_1), a_2 = a_1 \rightarrow a_2\}$. This system is not “positive” or “monotone” (cf. Problem 1) yet it is believed to be strongly normalizing. Note: As shown in [Mendler, 1991], positivity is a necessary and sufficient condition for strong normalization of a system defined by equations between simple types (without intersections).

References

[Mendler, 1991] Mendler, N. (1991). Inductive types and type constraints in the second-order lambda calculus. *Annals of Pure and Applied Logic*, 51(1-2):159–172.