TLCA List of Open Problems

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Problem # 13

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Statement. Inhabitation for intersection type systems

Problem Origin. Different variants of the problem were stated by Henk Barendregt, Mariangiola Dezani-Ciancaglini, Paula Severi, Paweł Urzyczyn, and others.

The inhabitation problem is to determine whether there exists a closed term of a given type. [Urzyczyn, 1999] shows the undecidability of inhabitation for the intersection type system of [Barendregt et al., 1983]. Decidable restrictions are discussed in [Kurata and Takahashi, 1995].

Many different intersection type systems have been introduced, mainly for describing λ-models: for a list see [Alessi et al., 2006] and the references there. A natural question is decidability of inhabitation for these systems. Using the notation of [Alessi et al., 2006], undecidability of the inhabitation for the system of [Barendregt et al., 1983] implies undecidability of the inhabitation for the systems $\mathcal{B}a$, $\mathcal{CDV}$, $\mathcal{E}n$. It is easy to show that all types are inhabited in $\mathcal{AO}$ and $\mathcal{P}a$ systems, since all terms inhabit the top type and all closed terms inhabit the bottom type [Honsell and Ronchi Della Rocca, 1992]. A similar argument shows decidability for $\mathcal{EHR}$ system. The question remains open for the systems $\mathcal{HL}$, $\mathcal{HR}$, $\mathcal{S}c$, $\mathcal{CDZ}$, $\mathcal{DHM}$.

A related issue is the inhabitation problem for intersection types of rank at most 3, according to the classification of [Leivant, 1983]. The undecidability proof in [Urzyczyn, 1999] works for rank 4, the problem for rank 2 is decidable, but EXPTIME-hard [Kuśmierek, 2007].

Yet another open question is the inhabitation problems for systems with recursive intersection types, cf. Problem 11.

Partial solution: The inhabitation problem turned out to be undecidable for types of rank 3 and EXPSPACE-complete for rank 2 [Urzyczyn, 2009].

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