

# TLCA List of Open Problems

<http://tlca.di.unito.it/opltlca/>

Updated February 4, 2014

## Problem # 14

*Submitted by* Simona Ronchi Della Rocca

*Date:* 1994

**Statement.** For every  $n$ , determine the set of terms which can be typed in the stratified polymorphic type assignment system of order  $n$ .

**Problem Origin.** First posed by Paola Giannini and Simona Ronchi Della Rocca

Giannini and Ronchi Della Rocca, in [Giannini and Ronchi Della Rocca, 1994], defined a complete stratification of the polymorphic type assignment system for  $\lambda$ -calculus, indexed by integers. The stratification of order  $n \geq 0$  is obtained by restricting the rule for eliminating the universal quantifier in the following way:

$$\frac{\Gamma \vdash_n M : \forall \alpha. \sigma \quad \text{the level of } \alpha \text{ in } \sigma \text{ is } \leq n}{\Gamma \vdash_n M : \sigma[\tau/\alpha]}$$

where the level of a variable in a type is defined as follows:

- i) if  $\alpha$  does not occur free in  $\sigma$ , then the level of  $\alpha$  in  $\sigma$  is 0;
- ii) the level of  $\alpha$  in  $\alpha$  is 1;
- iii) the level of  $\alpha$  in  $\forall \beta. \tau$  is the level of  $\alpha$  in  $\tau$ ;
- iv) the level of  $\alpha$  in  $\tau_1 \rightarrow \tau_2$  is  $\max\{n_1, n_2\} + 1$ , where  $n_1$  and  $n_2$  are respectively the levels of  $\alpha$  in  $\tau_1$  and  $\tau_2$  respectively.

It turns out that  $\vdash_0$  has the same typing power of the Curry type assignment system, and  $\vdash_1$  assigns types to all and only the normal forms. Find a characterization of the set of terms typable in the system indexed by  $n$ , for  $n \geq 2$ .

## References

[Giannini and Ronchi Della Rocca, 1994] Giannini, P. and Ronchi Della Rocca, S. (1994). A type inference algorithm for a complete stratification of the polymorphic type discipline. *Information and Computation*, 110:115 – 173.