

TLCA List of Open Problems

<http://tlca.di.unito.it/opltlca/>

Updated February 4, 2014

Problem # 16

Submitted by [Jakob Rehof](#)

Date: 1996

Statement. Is the subtype entailment problem decidable?

Problem Origin. First stated in [Pottier, 1996] and [Trifonov and Smith, 1996]. The present formulation is from [Henglein and Rehof, 1998].

We ask if the entailment problem with simple subtyping constraints over non-structurally ordered trees is decidable. Non-structurally ordered trees have a least element, \perp , and a greatest element, \top , which can be compared to any tree regardless of its tree domain (shape). Simple type expressions, τ , are finite terms built from \perp , \top and a binary constructor. Such expressions can be interpreted as denoting trees, and formal inequality constraints of the form $\tau \leq \tau'$ can consequently be valuated in the non-structural order on trees. For a finite set of constraints $C = \{\tau_i \leq \tau'_i\}_{i=1\dots n}$ of constraints and terms τ and τ' , we consider the entailment $C \models \tau \leq \tau'$, or, equivalently, validity of the first-order Horn implication $\forall \vec{\alpha}. (\bigwedge_{i=1}^n \tau_i \leq \tau'_i) \rightarrow \tau \leq \tau'$, where $\vec{\alpha}$ are the variables occurring in C , τ and τ' .

The problem first appears in slightly different forms in the papers [Pottier, 1996] and [Trifonov and Smith, 1996] for the purpose of simplifying subtyping constraints. It was studied in the form presented here in [Henglein and Rehof, 1998] (following the formulation in [Henglein and Rehof, 1997] for simple types), where it was shown to be to be PSPACE-hard. The full first-order theory of subtyping constraints has been shown to be undecidable [Su et al., 2002], but the question of decidability of entailment remains open [Rehof, 1998]. Further references can be found in [Rehof, 1998] and [Niehren and Priesnitz, 2003].

References

- [Dezani-Ciancaglini and Plotkin, 1995] Dezani-Ciancaglini, M. and Plotkin, G., editors (1995). *Typed Lambda Calculi and Applications*, volume 902 of *Lecture Notes in Computer Science*. Springer-Verlag.
- [Henglein and Rehof, 1997] Henglein, F. and Rehof, J. (1997). The complexity of subtype entailment for simple types. In *Logic in Computer Science*, pages 352–361. IEEE Computer Society Press.
- [Henglein and Rehof, 1998] Henglein, F. and Rehof, J. (1998). Constraint automata and the complexity of recursive subtype entailment. In *International Colloquium on Automata, Languages, and Programming*, volume 1443 of *Lecture Notes in Computer Science*, pages 616–627. Springer-Verlag.
- [Niehren and Priesnitz, 2003] Niehren, J. and Priesnitz, T. (2003). Non-structural subtype entailment in automata theory. *Information and Computation*, 186(2):319–354.
- [Pottier, 1996] Pottier, F. (1996). Simplifying subtyping constraints. In *International Conference on Functional Programming*, pages 122–133. ACM Press.

- [Rehof, 1998] Rehof, J. (1998). *The Complexity of Simple Subtyping Systems*. PhD thesis, Department of Computer Science, University of Copenhagen.
- [Su et al., 2002] Su, Z., Aiken, A., Niehren, J., Priesnitz, T., and Treinen, R. (2002). The first-order theory of subtyping constraints. In *Symposium on Principles of Programming Languages*, pages 203–216. ACM Press.
- [Trifonov and Smith, 1996] Trifonov, V. and Smith, S. (1996). Subtyping constrained types. In *Static Analysis Symposium*, volume 1145 of *Lecture Notes in Computer Science*, pages 349–365. Springer-Verlag.