TLCA List of Open Problems

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Updated February 4, 2014

Problem # 18

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Statement. Find trees representing contextual equivalences

Problem Origin. Stated by Mariangiola Dezani, Paula Severi and Fer-Jan de Vries.

If $\mathcal{P}$ is a set of λ-terms, the contextual equivalence $M \sim_{\mathcal{P}} N$ is defined by:

$$C[M] \longrightarrow M' \in \mathcal{P} \text{ if and only if } C[N] \longrightarrow N' \in \mathcal{P} \text{ for all contexts } C[\,],$$

where $\longrightarrow$ denotes β-reduction.

[Wadsworth, 1976] shows that $\sim_{\mathcal{H}}$, where $\mathcal{H}$ is the set of head normal forms, coincides with Böhm trees equality (up to infinite η's). [Hyland, 1976] shows that $\sim_{\mathcal{N}}$, where $\mathcal{N}$ is the set of normal forms, coincides with Böhm trees equality (up to finite η's).

The question is to find tree representations of λ-terms whose equalities coincide with the following contextual equivalences:

1. $\sim_{\mathcal{W}}$ where $\mathcal{W}$ is the set of weak head normal forms,
2. $\sim_{\mathcal{T}}$, where $\mathcal{T}$ is the set of top normal forms,
3. $\sim_{\mathcal{SA}}$, where $\mathcal{SA}$ is the set of strongly active terms,
4. $\sim_{\mathcal{SA}_X}$, where X is a set of λ-terms and $\mathcal{SA}_X$ is the set of strongly active terms depending on X,
5. $\sim_{\mathcal{HA}}$, where $\mathcal{HA}$ is the set of head active terms.

The set $\mathcal{T}$ is defined in [Berarducci, 1996], and the sets $\mathcal{SA}$, $\mathcal{SA}_X$, $\mathcal{HA}$ are defined in [Severi and de Vries, 2005, Kennaway et al., 2005].

References

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