

TLCA List of Open Problems

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Problem # 21

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Statement. Is higher-order matching decidable with many atoms?

It has recently been proved [Stirling, 2009] that the higher-order matching problem is decidable. However, the proof method only applies to the “classical” case of the problem: when all types are built from a single atom. Therefore, the problem remains open for the generalized case of types built from an arbitrary number of type variables.

A special case of the higher-order matching problem is the *retraction problem*, first mentioned in [de’Liguoro et al., 1992]. We say that a type ρ is a *retract* of a type τ (write $\rho \triangleleft \tau$) iff there exists a type environment Γ and terms

$$\Gamma \vdash F : \rho \rightarrow \tau \quad \text{and} \quad \Gamma \vdash G : \tau \rightarrow \rho$$

such that $G \circ F =_{\beta\eta} \mathbf{I}$. The problem to decide if $\rho \triangleleft \tau$ holds for given ρ and τ is decidable for a single atom [Padovani, 2001] but for many atoms it remains an open question. For a discussion see [Regnier and Urzyczyn, 2001].

Even less is known about polymorphic retractions. For instance, it is conjectured that $\rho \triangleleft \tau$ and $\tau \triangleleft \rho$ implies that ρ and τ are isomorphic. A confirmation of this conjecture yields a negative answer to Problem 17, see [Regnier and Urzyczyn, 2001].

References

- [de’Liguoro et al., 1992] de’Liguoro, U., Piperno, A., and Statman, R. (1992). Retracts in simply typed lambda-beta-eta-calculus. In *Logic in Computer Science*, pages 461–469. IEEE Computer Society Press.
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