TLCA List of Open Problems

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Problem # 22

Submitted by Furio Honsell *Date:* April 20 2007

Statement. Is there a continuously complete CPO model of the λ-calculus whose theory is precisely $\lambda\beta\eta$ or $\lambda\beta$?

Problem Origin. I asked myself this question in 1983. In 1984, on different occasions I asked it to Dana Scott and Gordon Plotkin. Both told me that they had already thought about it.

A CPO model is continuously complete if all Scott-continuous self-maps are represented by a point in the model. Continuously complete models are sometimes called retract models or reflexive models.

The theory of a model is the set of all equations between closed λ-terms which hold in the model. The problem can be phrased equivalently as:

- Is $\lambda\beta$ (or $\lambda\beta\eta$) the only equations which hold in all (extensional) continuously complete CPO models of λ-calculus? I.e. Are retract models complete for λ-calclulus?
- Is there a filter model whose theory is precisely $\lambda\beta\eta$ or $\lambda\beta$?

There are many related results, e.g.:

There exists a ω_1-continuously complete ω_1-CPO model whose theory is precisely $\lambda\beta\eta$ [Di Gianantonio et al., 1995].

There exist theories which do not have continuously complete CPO-models [Honsell and Ronchi Della R

References

[Di Gianantonio et al., 1995] Di Gianantonio, P., Honsell, F., and Plotkin, G. (1995). Uncountable limits and the lambda calculus. *Nordic Journal of Computing*, 2(2):126–145.

[Honsell and Ronchi Della Rocca, 1992] Honsell, F. and Ronchi Della Rocca, S. (1992). An approximation theorem for topological lambda models and the topological incompleteness of lambda calculus. *Journal of Computer and System Sciences*, 45(1):49–75.