TLCA List of Open Problems

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Problem # 23

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Statement. Does Expansion Postponement hold for every PTS?

Problem Origin. This property was first conjectured by Pollack [Pollack, 1992] .

We can replace the usual conversion rule in a Pure Type System (PTS):

$$\frac{\Gamma \vdash M : A \qquad \Gamma \vdash B : s \qquad A =_\beta B}{\Gamma \vdash M : B}$$

with the following two rules:

$$\text{(reduction)}\ \frac{\Gamma \vdash M : A \qquad A \twoheadrightarrow_\beta B}{\Gamma \vdash M : B}$$

$$\text{(expansion)}\frac{\Gamma \vdash M : B \qquad \Gamma \vdash B : s \qquad B \twoheadrightarrow_\beta A}{\Gamma \vdash M : A}$$

Expansion Postponement (EP) is the conjecture that every derivable judgement in a PTS has a derivation in which the expansion rule is used at most once, as the final step in the derivation.

This was proposed by Pollack in 1992 [Pollack, 1992], where he showed that it is a necessary condition for the type-checking algorithm given there to be complete. Gutiérrez and Ruiz [Gutiérrez and Ruiz, 2003] proved that it is a necessary condition for cut elimination in a sequent calculus formulation of PTSs.

It was shown by Erik Poll [Poll, 1998] to hold for every weakly normalising PTS, and by Barthe [Barthe, 1998] to hold for injective PTSs.

References

[Barthe, 1998] Barthe, G. (1998). Type-checking injective pure type systems. *Journal of Functional Programming*, 9:675–698.

[Gutiérrez and Ruiz, 2003] Gutiérrez, F. and Ruiz, B. (2003). Cut elimination in a class of sequent calculi for pure type systems. *Electronic Notes in Computer Science*, 84:105–116.

[Poll, 1998] Poll, E. (1998). Theoretical pearl: Expansion postponement for normalising pure type systems. *Journal of Functional Programming*, 8:89–96.

[Pollack, 1992] Pollack, R. (1992). Typechecking in Pure Type Systems. In *Informal Proceedings of the 1992 Workshop on Types for Proofs and Programs, Bastad, Sweden*, pages 271–288.