

TLCA List of Open Problems

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Updated February 4, 2014

Problem # 24

Submitted by Corrado Böhm

Date: 2013

Statement. On the equational meaning of deeds

Problem Origin. The problem has been posed first by Corrado Böhm. It is added to the list to celebrate his 90th birthday.

A deed is a closed normal form with only one initial abstraction, namely a deed has shape $\lambda x.xP_1 \dots P_n$ where $n \geq 0$.

It is easy to verify that each equation of the shape

$$D_1X = D_2X,$$

where D_1, D_2 are deeds, can be solved, for instance by choosing a fixed point of the combinator K as X .

It is possible to define a one-one mapping between arbitrary normal forms and deeds, as follows. Given a normal form M with $m \geq 0$ initial abstractions, the deed corresponding to M is:

$$D^M \equiv \lambda x.M(xc_1) \dots (xc_m),$$

where c_1, \dots, c_m are the Church numerals or any distinct closed beta-eta normal forms. Note that the mapping is injective by Böhm's theorem.

The question is as follows: What can we understand about the solution of the equation $MX = NX$, where M, N are arbitrary normal forms, by looking at the equation $D^MX = D^NX$?

References