TLCA List of Open Problems

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Problem # 4

Submitted by Richard Statman *Date:* 1985

Statement. A fixed point combinator from B and ω alone?

Problem Origin. The problem was first posed by Raymond Smullyan.

The combinator ω satisfies the reduction rule $\omega x \Rightarrow xx$ and the combinator B is as usual $\mathsf{B}xyz \Rightarrow x(yz)$. Is there an applicative combination of B and ω which is a fixed point combinator? The problem was originally proposed in [Smullyan, 1985].

Comments by Richard Statman: Note that $\omega(\mathsf{B}x\omega)$ is always a fixed point of x but it appears that x cannot be abstracted to give a fixed point combinator Y such that $\mathsf{Y}x = x(\mathsf{Y}x)$. If it is required that $\mathsf{Y}x \twoheadrightarrow x(\mathsf{Y}x)$ as with Turing's fixed point combinator then it can be shown [Statman, 1993, McCune and Wos, 1991] that no B,ω combination works.

References

[McCune and Wos, 1991] McCune, W. and Wos, L. (1991). The absence and the presence of fixed point combinators. *Theoretical Computer Science*, 87:221–228.

[Smullyan, 1985] Smullyan, R. (1985). *To Mock a Mockingbird*. Knopf.

[Statman, 1993] Statman, R. (1993). Some examples of non-existent combinators. *Theoretical Computer Science*, 121:441–448.