TLCA List of Open Problems

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Problem # 5

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Statement. Are there terms untypable in F_ω but typable with help of positive recursive types?

Consider an extension of simply typed lambda-calculus obtained by adding the construct $\mu p\,\tau$ for types τ containing only positive occurrences of p, together with the rules:

$$\frac{\Gamma \vdash M : \tau[p{:=}\mu p\,\tau]}{\Gamma \vdash M : \mu p\,\tau} \qquad\qquad \frac{\Gamma \vdash M : \mu p\,\tau}{\Gamma \vdash M : \tau[p{:=}\mu p\,\tau]}$$

This system can assign types for terms untypable in F, e.g. the combinator **22K** is typable, where **2** is the Church numeral [Urzyczyn, 1996]. Are there terms typable in this system that cannnot be typed in F_ω? Note that **22K** can be typed in F_ω [Urzyczyn, 1997]. Note also that $\lambda x.\, xx$, easily typable in system F, is clearly untypable with positive recursive types.

References

[Urzyczyn, 1996] Urzyczyn, P. (1996). Positive recursive type assignment. *Fundamenta Informaticae*, 28(1-2):197–209.

[Urzyczyn, 1997] Urzyczyn, P. (1997). Type reconstruction in F_ω. *Mathematical Structures in Computer Science*, 7(4):329–358.