

# TLCA List of Open Problems

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## Problem # 6

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**Statement.** Basis Decision Problem.

The *basis decision problem*, originally stated in [Statman, 1993], is to decide whether a given finite set of proper combinators is combinatorially complete. (A *proper* combinator [Barendregt, 1984, p. 184] is one of the form  $\lambda\vec{x}. M$ , where  $M$  contains no lambda, and  $FV(M) \subseteq \vec{x}$ .) It follows from Theorem 4 in [Statman, 1986] that the problem is undecidable for finite sets of normal (possibly improper) combinators. A good solution would be an abstraction algorithm, parameterized by a given set of combinators, which returned an error message on non-bases. Broda and Damas have proved that in the linear case the problem is decidable [Broda and Damas, 1997].

## References

- [Barendregt, 1984] Barendregt, H. (1984). *The Lambda Calculus. Its Syntax and Semantics*. North-Holland, second edition.
- [Broda and Damas, 1997] Broda, S. and Damas, L. (1997). On combinatory complete sets of proper combinators. *Journal of Functional Programming*, 7(6):593–612.
- [Statman, 1986] Statman, R. (1986). On translating lambda terms into combinators. In *Logic in Computer Science*, pages 378–382. IEEE Computer Society Press.
- [Statman, 1993] Statman, R. (1993). Some examples of non-existent combinators. *Theoretical Computer Science*, 121:441–448.