TLCA List of Open Problems

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Problem # 7

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Statement. Word problem for combinators of orders less than 3.

The *word problem* for a set of combinators C is to determine if two applicative combinations of members of C are $\beta\eta$-equal. A proper combinator A, with reduction rule $\mathsf{A}x_1 \ldots x_n \Rightarrow X$, where X is an applicative combination of $x_1, \ldots, x_n$ is said to have *order n*. We ask if the word problem for all proper combinators of orders less than 3 is decidable. The problem was originally proposed in [Statman, 1989].

Comments by Richard Statman: It is easy to see that the word problem for combinators of order 1 is decidable. More can be shown [Statman, 1988a]. In [Statman, 1989] it is shown that the word problem for Smullyan's Lark combinator is solvable even though fixed points always exist. In [Statman, 2000] we prove that the word problem for all co-compositors and finitely many instances of (finitely many) compositors is solvable. We have been led to the conjecture: The word problem for all proper combinators of order less than 3 (taken together in one system) is decidable. Some combinator of order 3 or more is necessary for a basis of proper combinators [Statman, 1986] but slightly beyond order 2 [Statman, 1988b] leads to undecidabilty.

Added "in proof": This problem also occurs in the <u>RTA LOOP</u> as problem number 96.

References

[Statman, 1986] Statman, R. (1986). On translating lambda terms into combinators. In *Logic in Computer Science*, pages 378–382. IEEE Computer Society Press.

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[Statman, 1988b] Statman, R. (1988b). Combinators hereditarily of order two. Technical Report 88-33, Department of Mathematics, Carnegie Mellon University.

[Statman, 1989] Statman, R. (1989). The word problem for Smullyan's Lark combinator is decidable. *Journal of Symbolic Computation*, 7(2):103–112.

[Statman, 2000] Statman, R. (2000). On the word problem for combinators. In *Rewriting Techniques and Applications*, volume 1833 of *Lecture Notes in Computer Science*, pages 203–213. Springer-Verlag.